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Numerical simulation of non-invasive determination of the propagation coefficient in arterial system using two measurements sites

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Abstract. Literature shows a lack of works based on non-invasive methods for computing the propagation coefficient γ, a complex number related to dynamic vascular properties. Its imaginary part is inversely related to the wave speed C through the relationship $C = \omega/\text{Im}(\gamma)$, while its real part a, called attenuation, represents loss of pulse energy per unit of length. In this work an expression is derived giving the propagation coefficient when assuming a pulsatile flow through a viscoelastic vessel. The effects of physical and geometrical parameters of the tube are then studied. In particular, the effects of increasing the reflection coefficient, on the determination of the propagation coefficient are investigated in a first step. In a second step, we simulate a variation of tube length under physiological conditions. The method developed here is based on the knowledge of instantaneous velocity and radius values at only two sites. It takes into account the presence of a reflection site of unknown reflection coefficient, localised in the distal end of the vessel. The values of wave speed and attenuation obtained with this method are in a good agreement with the theory. This method has the advantage to be usable for small portions of the arterial tree.

PACS. 47.15.-x Laminar flows – 47.90.+a Other topics in fluid dynamics

1 Introduction

The waves generated by the heart, propagate along the arterial system. Their form changes due to the properties of artery itself, and the reflections at singularities. To describe the contribution of arterial properties on pressure — or velocity — wave propagation, the complex propagation coefficient γ has to be calculated. When considering an unidirectional propagating wave of an hemodynamic variable ϕ, this coefficient can be introduced by writing in a complex formulation:

$$\phi = A\exp(i\omega t - \gamma x) = A\exp\left(i\omega\left(t - \frac{x}{C}\right)\right)\exp(-ax)$$
$$\text{with} \quad \gamma = a + i\frac{\omega}{C}.$$

In this expression $a > 0$ is the attenuation $C > 0$ is the phase velocity and x the longitudinal coordinate. The determination of propagation coefficient is of a great importance. Attenuation is related to the energy dissipation due to the blood viscosity, viscoelasticity of the wall, reflection in bifurcations and disease sites as atherosclerosis or occlusion [1]. The imaginary part of propagation coefficient is inversely proportional to phase velocity that represents a parameter of clinical interest [2,3]. Several authors use the pulse wave velocity, calculated from the Moens-Korteweg equation, to derive arterial distensibility and Young's modulus. It is strongly correlated to the rheological properties of vessels walls [4,5]. Other authors use the pulse wave velocity as a mean to estimate stiffness of the vessel walls [6,7] and arterial distensibility [8,9]. The knowledge of the wave speed is also necessary to separate the arterial wave pressure and flow into their backward and forward running components [10–13]. Other authors use the phase velocity as an index of arterial health [14, 15]. The measurement of the propagation coefficient has been widely used as a mean of assessing the severity of a disease as atherosclerosis [16]. For the determination of phase velocity, two ways are used in the literature:

– the first way assumes that the wall mechanical properties are known. It uses two formulas:
the Moens-Korteweg equation $c = \sqrt{Eh\,/\,2\rho R}$
the Womersley-Jager equation $\gamma = \sqrt{Z_l Y_t}$
where E is the Young's modulus of the tube, R its radius, h its wall thickness, ρ the fluid density, Z_l the longitudinal impedance and Y_t the transverse admittance;
– the second way is based on the measurement of the pulse wave travelling along the cardiovascular system. In the time domain, the phase velocity has been

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evaluated as the ratio of distance d between the two sites of measurement and the associated transit time. This method is inaccurate due to the difficulties in defining the foot of the wave [17]. Applying Fourier-analysis Milnor [18] calculates an apparent phase velocity, C_{app} for each harmonic of the signal (frequency f and phase lag $\Delta\phi$):

$$c_{app} = \frac{2\pi f \Delta x}{\Delta\phi}.$$

A new method named "PU-loop" for Pressure Velocity relationship, derived by Khir et al. [10], has been used to compute the wave speed, and tested by the authors in presence of arterial occlusion [19–21]. Rabben et al. [22] derived an other method to compute the phase velocity assuming the knowledge of radius and flow rate. Other authors [23,24] used the propagation coefficient determined by two or three points method to compute the phase velocity $c = \omega/\mathrm{Im}(\gamma)$. This formula will be used in this work to compute C_c, a so called "true phase velocity".

The determination of the propagation coefficient in arterial system has been the subject of many researches in the last few decades. In absence of significant reflection or when the reflection coefficient of the distal site was known (equal to unity), a two-point method based on the measurement of pressure or flow in two sites [25] can be used allowing the derivation of the true propagation coefficient. Investigators using, two-points method [26,27] found values in agreement with values calculated by Womersley theory. The accuracy of this two-point method is accepted, but it has a limited application: it assumed a known reflection coefficient produced by a total occlusion, which is not the case in most hemodynamic conditions, when the lumen of artery is partially occluded.

Without an assumption about one of the parameters and taking into account reflections from local vascular sites, Taylor developed a method known as three-point method based on the measurements of pressure or flow in three equidistant sites of the vessel of interest. The results obtained by some authors [28,29] using three points method, measuring pressures at three sites, are not in a good agreement with theoretical Womersley's predictions. Phase velocity found by these investigators was 20% lower, and the attenuation coefficient was about eight times higher. In other researches, an agreement was found between theoretical and experimental propagation coefficient values, for uniform tube [17,30].

In case of an unknown reflection coefficient, a second approach using four transducers, requiring two measurements of pressure and flow at two sections of the vessel investigated, has been described by Milnor and Nichols [31], and used by Milnor and Bertram [32] to measure propagation coefficient in the femoral and carotid artery. Bertram et al. [23] described, an iterative general method, for calculating propagation coefficient using two pressure measurements, one flow rate and one diameter vessel. This method has been used in vivo and in vitro to compute the propagation coefficient [30] and attenuation [26].

Reasons for discrepancy between several literature studies are still unclear. Busse et al. [33] explained this discrepancy as a result of intrinsic inaccuracy in experimental techniques. Li et al. [28] explained these differences as a result of neglecting non linear terms in Womersley theory. Reuderink et al. [17] demonstrated that geometrical taper affects the values of propagation coefficient given by the three-point method only at given frequencies. Moreover, damping is decreased by tapering instead of being increased, as found in literature works. This study shows that for a uniform tube, the celerity obtained by the three-point method is in a good agreement with that calculated using Womersley theory. This method is confronted with another difficulty in vivo, in the fact that many arteries are short and the method requires twice the distance between the transducers. This distance must be more than a critical length (d) to obtain measurement with adequate signal-to-noise ratio. In recent studies of propagation coefficient, Bertram et al. [30] showed that propagation coefficients obtained by several techniques were similar; they concluded that the discrepancies between studies in literature can not be due to problems associated with the methods themselves but caused by variations in experimental conditions or other unknown artefacts.

As we have shown, there is several factors of uncertainty in the determination of propagation coefficient and the literature shows a lack of works based on non-intrusive methods for the determination of this parameter of clinical interest. Propagation coefficient has been determined only invasively. With the aim of solving this problem we propose in this work a non-invasive method allowing the determination of the propagation coefficient, based on ultrasound measurements of velocities and radius at two sites separated with a known distance (d). This methods requires no assumption about the reflection coefficient conversely to major literature works using two-points method. We will critically re-examine the exactitude of the method on a determination of waves speed and attenuation for different hemodynamic conditions, using numerical simulation. In particularly we will study, the effects of increasing the reflection coefficient on a true determination of propagation coefficient (attenuation and wave speed) in first time. In second time, we will examine the applicability of the method to physiological conditions by simulation of several lengths. The effect of noise has also been investigated to simulate experimental conditions.

2 Mathematical model

Consider pulsate, laminar flow through a uniform, viscoelastic and impermeable vessel of instantaneous radius $R(x,t)$, of length L, terminated by an equivalent site of reflection, with reflection coefficient K.

The fluid is assumed to be Newtonian, incompressible; the viscosity is taken to be constant, the effect of gravity is negligible. The velocity of the fluid enclosed within the cylindrical coordinates is denoted by $V = [u(r,x,t), v(r,x,t)]$, where r is the radial coordinate, x is the position along the vessel, t is time, u the radial velocity and v the axial velocity.

We assume that parietal deformations are small as compared to the radius R, and that R is small as compared to the wavelength; Moreover, the fluid velocity is small as compared to wave speed $v \ll C$. Each hemodynamic parameter can be expressed in the form:

$$\Phi(r,x,t) = \sum_{-\infty}^{\infty} \phi(r,x,\omega_n) \exp i\omega_n t.$$

We also assumed that the wall of the vessel only undergoes radial motions.

The flow is assumed to be axisymmetric:

$$u(0,x,t) = 0 \quad \text{and} \quad \left.\frac{\partial v(R,x,t)}{\partial r}\right|_{r=0} = 0. \tag{1}$$

The no-slip condition is satisfied (the velocity of fluid at the wall equals the velocity of the wall):

$$v(R,x,t) = 0 \quad \text{and} \quad u(R,x,t) = \frac{\partial R}{\partial t}. \tag{2}$$

The pressure is quasi constant over the cross-sectional area of the blood vessel. With these hypothesis the convective terms in the Navier Stokes equations can be neglected in accordance to the Womersley theory [41], and the governing equations given in the following form:
The continuity equation

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial v}{\partial x} = 0. \tag{3}$$

In the axial direction

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \eta\left[\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r}\right]. \tag{4}$$

In the radial direction

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \eta\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right]. \tag{5}$$

One makes the assumption of the presence of a single equivalent site of reflection for this part of the arterial trunk (fictive region representing the whole of the reflections of the terminal sites), located at a distance $x = L$ from the origin of the tube, and η is the blood kinematics viscosity.

Using the linear theory of propagating waves as introduced by Womersley [40], Flaud et al. [13] gave an expression of the centre line velocity $V_{cln}(x)$ and of the radius $R_n(x)$ for an harmonic n:

$$V_{cln}(x) = V^f_{cln}(x=0)\,(\exp(-\gamma x) - K\,\exp(-\gamma(2L-x))) = V^f_{cln}(x) + V^b_{cln}(x) \tag{6}$$

$$R_n(x) = R^f_n(x=0)\,(\exp(-\gamma x) + K\,\exp(-\gamma(2L-x))) = R^f_n(x) + R^b_n(x) \tag{7}$$

where, for a wavelength large as compared to the radius of the tube or $R\gamma_n \ll 1$,

$$V^f_{cln}(x=0) = \frac{\gamma_n P_n}{i\omega_n \rho}\left(1 - \frac{1}{J_0(z_2)}\right) \tag{8}$$

$$R^f_n(x=0) = \frac{\gamma_n^2 R\,P_n}{2\omega_n^2 \rho}\left(\frac{2J_1(z_2)}{z_2\,J_0(z_2)} - 1\right). \tag{9}$$

In these expressions the upperscript f (respectively b) stands for an unidirectional forward (respectively backward) propagating wave, the subscript n for the rank of the harmonic of the Fourier analysis, cl for the centre measurement. P_n is the pressure, J_0 and J_1 are the zero and first order of the first kind Bessel function, $z_2 = \alpha_n\, i^{3/2}$ and $\alpha_n = R\sqrt{\omega_n / \eta}$ is the Womersley parameter associated to the pulsation ω_n.

2.1 Theory: computation of the propagation coefficient

Eliminating $\exp(-\gamma x)$ and $K\,\exp(-\gamma(2L-x))$ from (6) and (7), one can write:

$$2V^f_{cln}(x) = V^f_{cln}(x=0)\left(\frac{V_{cln}(x)}{V^f_{cln}(x=0)} + \frac{R_n(x)}{R^f_n(x=0)}\right) \tag{10}$$

$$= V_{cln}(x) + R_n(x)\,\frac{V^f_{cln}(x=0)}{R^f_n(x=0)} = V_{cln}(x) + R_n(x)\,\frac{H}{\gamma_n} \tag{11}$$

where

$$\frac{H}{\gamma_n} = \frac{1}{\gamma_n}\left(\frac{2i\omega_n}{R}\,\frac{z_2(1 - J_0(z_2))}{2J_1(z_2) - z_2 J_0(z_2)}\right). \tag{12}$$

Assuming the measurement of radius and centre line velocity at $x = x_1$ and $x = x_2$, the measured quantities $R_n(x_1)$, $R_n(x_2)$, $V_{cln}(x_1)$ and $V_{cln}(x_2)$ are now known, and since

$$V^f_{cln}(x_2) = V^f_{cln}(x_1)\,\exp(-\gamma_n\,d). \tag{13}$$

We can write, using equations (10)–(13) for sites (1-2):

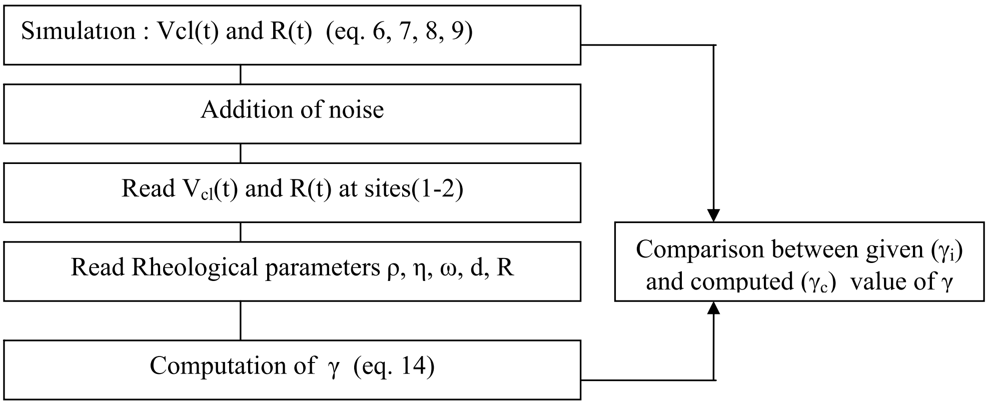
$$\frac{V^f_{cln}(x_2)}{V^f_{cln}(x_1)} = \exp(-\gamma_n\,d) = \frac{V_{cln}(x_2) + R_n(x_2)\,H/\gamma_n}{V_{cln}(x_1) + R_n(x_1)\,H/\gamma_n}. \tag{14}$$

Assuming $\gamma_n d \ll 1$, which is coherent since d is of the order of a few R, we can develop $\exp(-\gamma_n d)$ in the vicinity of zero and we obtain with a third order development:

$$\gamma_n^4\frac{V_{2n}d^3}{6} + \gamma_n^3\left(\frac{V_{2n}d^2}{2} + \frac{HR_{2n}d^3}{6}\right) + \gamma_n^2\left(V_{2n}d + \frac{HR_{2n}d^2}{2}\right) + \gamma_n\,(V_{2n} - V_{1n} + R_{2n}Hd) + (R_{2n} - R_{1n})\,H = 0 \tag{15}$$

where $V_{in} = V_{cln}(x_i)$, $R_{in} = R_n(x_i)$, $i = (1,2)$. This fourth degree equation shows that the determination of

Table 1. Simplified representation of the numerical procedure.



propagation coefficient is possible, starting from the measurement of instantaneous radius and velocities in two sections of an arterial tree. The physical solution of such an equation is obtained thanks to the condition $a > 0$ and $C > 0$. The diameter of vessel can be obtained by echo-tracking [13], blood velocity can be also measured by ultrasound Doppler techniques [4].

2.2 Parameters and numerical model

In order to validate this method we present a numerical simulation using theoretically computed velocity and radius. We consider a uniform tube of finite length equal to 'L', loaded with a distal resistance 'K' and filled with viscous fluid as shown in Figure 1 (top). A single impulse of fundamental frequency $F_0 = 1$ Hz, was generated in the tube. The number of harmonic constituting the signal was equal to 15 harmonics. The wave speed of velocity and radius is 'C'. The attenuation is equal to 'a'. The distance between transducers was chosen to be equal to $d = 3$ cm. Sinusoidal velocities and radius signals of different frequencies were used for the simulation. To respect the assumptions of linearity we have chosen an amplitude of velocity signal lower than 0.5 m/s and an amplitude of radius displacements lower than 0.4×10^{-3} m.

We assumed the following parameters values as input values: $L = 43.3$ cm; $d = 3$ cm; $\rho = 1.05 \times 10^3$ kg/m^3; $\eta = 3.3 \times 10^{-6}$ m^2 s^{-1}; $x_1 = 5$ cm; $a_i = 0.4$ m^{-1}.

The numerical computation can be summarised in the following way (Tab. 1):

1/ Simulation: for a given set of values of γ, K, ω_0, P_n, ρ, R_0, η, L, x_1, and x_2, and using equations (6), (7), (8) and (9) we compute the simulated velocity and radius at $x = x_1$, and $x = x_2 = x_1 + d$.

2/ Noise: to take into account the experimental noise, we added noise to velocity and radius.

3/ These noised quantities are now used, with the values of K, ω_0, ρ, R_0, η, and d to resolve equation (15) and compute γ_c which is compared to the initial given value γ_ι.

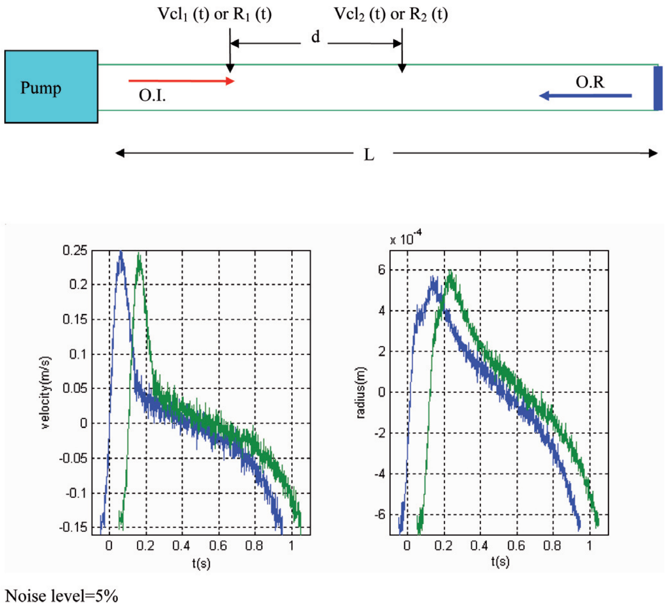


Fig. 1. (Color online) Top panel shows schematic diagram of the strained tube with a site of reflection located at $x = L_0$. used for the simulation of waves, d was the distance between transducer. Bottom panel shows examples of the velocities (left) and radius (right), noisy waveforms (5%), in site 1 (blue), site 2 (green) used on simulation.

3 Results

Bottom panel of Figure 1 shows examples of the velocities (left) and radius (right), waveforms with noise of 5%.at the two different sites used in the simulation: site 1 (blue), site 2 (green). The first result is presented Figure 2 for noiseless signals. The normalised phase velocity C_n (i.e. the computed phase velocity C_c divided by the input value C_i) is computed at each frequency between 1 and 15 Hz, in a viscoelastic tube of length L, for low reflection coefficient (top panels) and high reflection coefficient (bottom panel). We simulated two distances between the transducers: a small distance ($d = 3$ cm at left) or a large distance ($d = 8$ cm, right). In each panel we plotted in green the "apparent phase velocity" derived by assuming

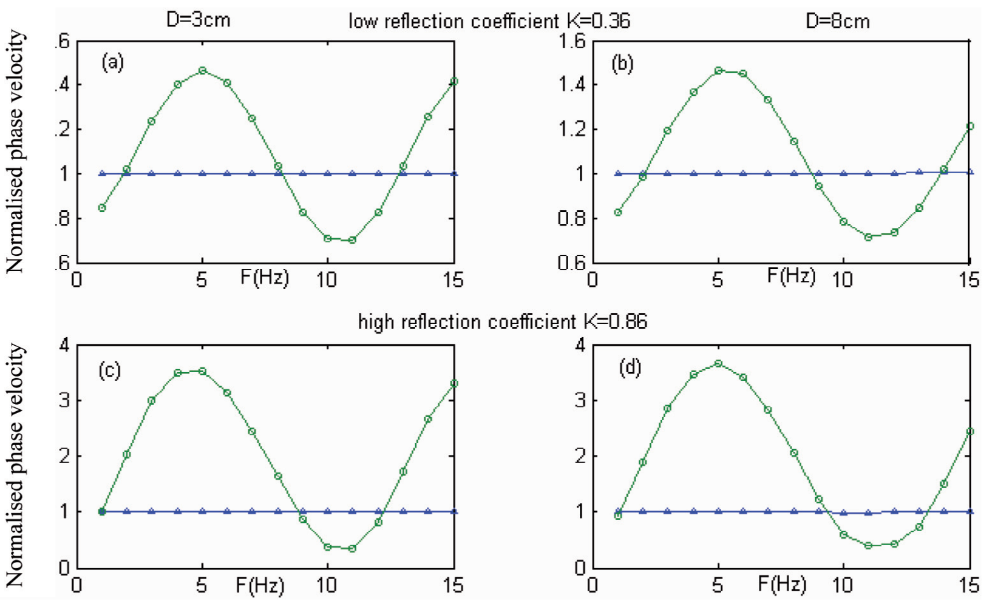


Fig. 2. Normalized phase velocity (computed phase velocity divided by the input value of C) obtained by RV-method: without noise (Δ), normalised "apparent phase velocity" ($\circ$), for uniform viscoelastic tube, of length $L = 43.3$ cm at low reflection coefficient $K = 0.36$ (top panels) and high reflection coefficient $K = 0.86$ (bottom panels). The panel in the left column show data obtained for ($d = 3$ cm), the panel in the right column show data obtained for ($d = 8$ cm).

an unidirectional propagation and the actual phase velocity (in blue) computed by the RV-two point method for the same physical and geometrical parameters. Figure 3 shows the results giving the normalised attenuation a_n (i.e. the computed attenuation a_c divided by the input value of a, a_i) over the range of frequency in a similar manner as in Figure 2. Figures 2 and 3 show a very good agreement between simulated and theoretical wave speed and attenuation. Values obtained are close to theoretical values at low and high reflection coefficient. While "apparent wave speed" and attenuation show an oscillatory behaviour over the range of frequency investigated. The amplitude of oscillation of apparent phase velocity is greater at high frequency. It is one and half time the phase velocity and ten time the attenuation obtained by RV-two point method at low reflection coefficient ($K = 0.36$). Moreover the amplitude of "apparent wave speed" and attenuation increases with the augmentation of the reflection coefficient. We can see, in Figure 2 (bottom) that the apparent phase velocity is about three and an half times the theoretical value which is quasi equal to the wave speed obtained by RV-two point method. Figure 3 (bottom) shows an "apparent attenuation" of fourteen times the theoretical value. The phase velocity and attenuation, derived from our method are in a good agreement with theoretical values for low and high reflection coefficient.

Figure 4 shows the results for simulated normalised phase velocity (left) and normalised attenuation (right) in the case of noisy signals (noise level = 0%, 2% and 5%). The computed values oscillated slightly about the theoretical values. The amplitude of oscillation, increases with frequency for both phase velocity and attenuation, while the discrepancy is greater for attenuation. Increasing the amplitude of noise, affects the determination of the true propagation coefficient.

Figure 5a shows three dimensional frequency pattern of wave speed, between 1 and 10 Hz for different values of reflection coefficient, which increases from 0.06 to 0.96. The phase velocity values computed by RV-two point method, are in a close agreement with theoretical values over the investigated range of frequency. The augmentation of reflection coefficient has no effect on the determination of the true phase velocity.

Figure 5b shows three dimensional frequency pattern of normalised attenuation, between 1 and 10 Hz for different values of the reflection coefficient, which increase from 0.06 to 0.96. Attenuation computed on all sections, shows similar behaviour. An increase of the reflection coefficient has no effect on the determination of wave speed computed by the RV-method. The deviation of attenuation values computed by the RV-method over the range of frequency is very small; it varies between 1.001 and 0.999 times the theoretical value for all values of reflection coefficient used over the investigated range of frequency.

The deviations of phase velocity and attenuation, are small compared to the theoretical values, the maximum of wave speed obtained by RV-method over the range of frequency investigated is 1.001 times the theoretical values; and the minimum was about 0.999 times the theoretical phase velocity.

Figure 6a shows three dimensional frequency pattern of normalised wave speed, between 1 and 10 Hz for different values of the position of the equivalent site of reflection, which vary between $L = 16$ cm and $L = 96$ cm. The simulated values of wave speed are in a good agreement with theoretical values over the investigated range

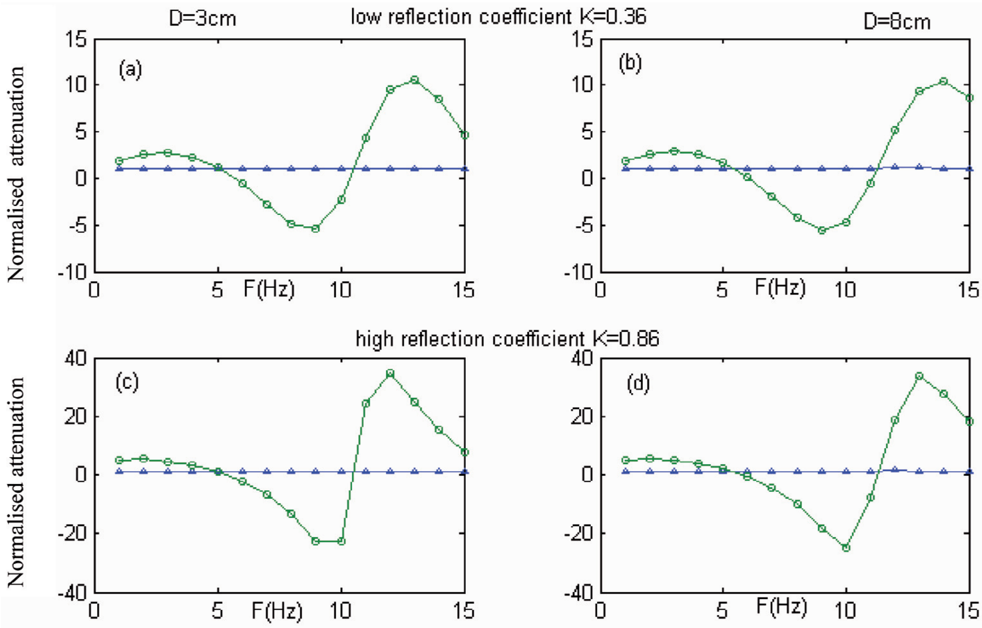


Fig. 3. Normalized attenuation (computed attenuation divided by the input value of a) obtained by the RV-two point method: without noise (Δ), normalized "apparent attenuation" (o), for uniform viscoelastic tube, of length $L = 43.3$ cm. The legend panels are the same as in (Fig. 2).

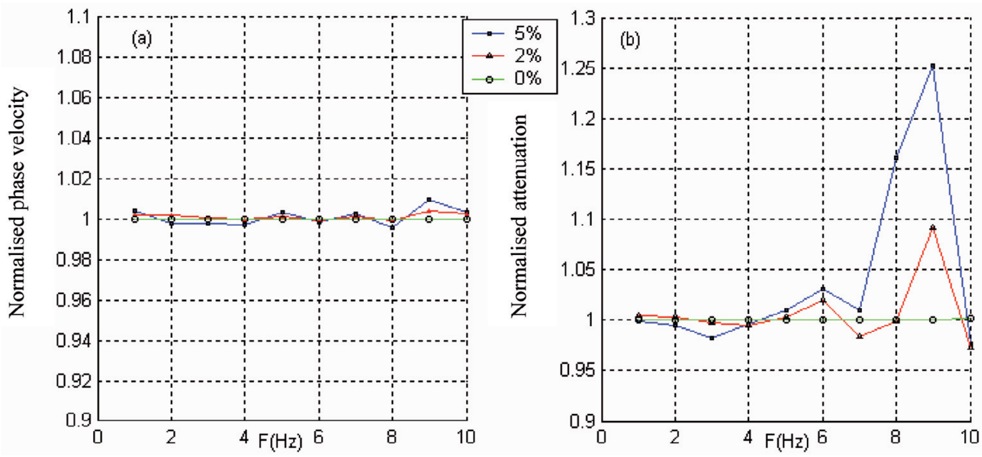


Fig. 4. Frequency patterns of normalised phase velocity (a, left) and normalised attenuation (b, right) for different amplitude of noise (0%, 2%, and 5%).

of reflection coefficients while some discrepancy can be observed at high frequency. The deviations of the phase velocity values computed by the RV-two points method at high frequency are small. They are about 1% of the theoretical values.

Figure 6b shows a three dimensional frequency pattern of normalised attenuation, between 1 and 10 Hz in the same manner as in Figure 5. The deviations at high frequency are greater than that observed for wave speed.

4 Discussion

The aim of our study was to present a theoretical linear method, for estimating propagation coefficient in arterial vessels, to validate the method by simulation and investigate the effect produced by changing different parameters on the determination of propagation coefficient. This method, based on four arterial wave form measurements, measurement of radius and centre line velocity at two arterial sites, can be applied non-invasively using Doppler techniques. Moreover linear analysis, has the advantage that the system of equations can be solved in the frequency domain with taking into account the viscoelastic behaviour of the vessel wall

The assumption of linear arterial system is violated in great arteries as in aorta where Milnor et al. [18], found values of peak velocities representing a significant fraction of phase velocity. Arnold et al. [34] and Reneman et al. [35] have shown that radial displacements

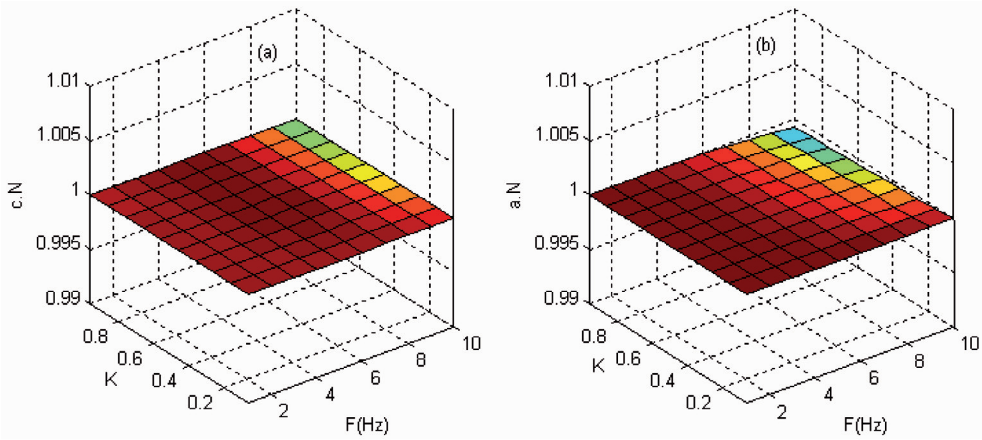


Fig. 5. Three dimensional view of frequency pattern of normalized phase velocity (a, left) and normalised attenuation (b, right) for different reflection coefficient values (from 0.06 to 00.96 by steps of 0.10).

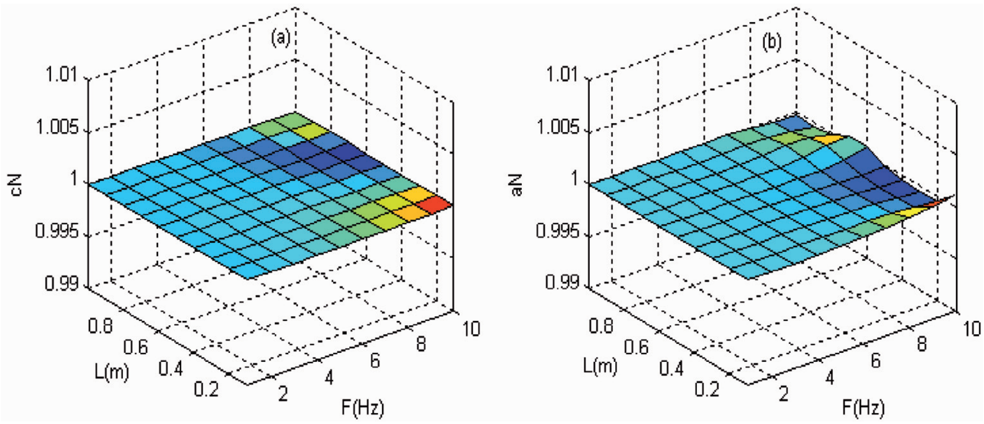


Fig. 6. Three dimensional view of frequency pattern of normalized phase velocity (a, left) and normalised attenuation (b, right), for different tube lengths. Reflection coefficient $K = 0.56$, attenuation $a = 0.4$ m^{-1} and theoretical phase velocity $C = 12$ m/s.

of human great arteries are about 10% of the radius. Learoyd and Taylor [36] showed a non-linear pressure-radius relationship. Nevertheless, several studies found that linear model, where we can include easily wall viscoelastic properties, seems to be more appropriate than non linear model: Comparatives studies [24,37], using non linear or linear model including viscoelastic wall behaviour, showed an agreement between linear and non linear response. Phytoud et al. [12] concluded that linear analysis would suffice for most clinical purposes. The linear model has the advantage that it is easier to solve accurately the governing equation of fluid and wall than with non linear model.

The fundamental result arising from our work is that linear model, including wall viscoelasticity, gives good predictions of phase velocity and attenuation for all reflections coefficient values over the whole range of frequencies investigated (1–10 Hz), in particularly at low frequency as shown in Figure 5a corresponding to phase velocity and Figure 5b corresponding to the attenuation. The phase velocity and attenuation, obtained are close to the theoretical values for non noisy radius and velocities signals.

The computed values of phase velocity and attenuation fluctuated slightly around the theoretical values at low frequency when we add noise of 5% to radius and velocities (Figs. 4a and 4b). The deviations are greater at high frequency, which can be easily explained by the decrease of the signal-to-noise ratio. Even in the absence of noise (Fig. 2) the apparent phase velocity shows an oscillatory behaviour ; its magnitude increases with increasing the value of the reflection coefficient over the investigated range of frequency. Similar behaviour has been found in experimental works done by Reuderink et al. [17]. The same conclusion can be deduced for the attenuation as shown in Figure 3.

Comparison between Figures 2a and 2b, which correspond to the same physical and geometrical tube parameters, but for different distances between the sites of measurements, allow to conclude that phase velocity determination is quasi independent (without noise) on distance between transducers. This finding is in agreement with experimental conclusion derived by Ursino et al. [27]. The same conclusion can be derived for attenuation as shown in Figure 3.

The results obtained at two values of reflection coefficient (top and bottom panels of Figs. 2 and 3) for the same length, show that the "apparent propagation coefficient" is strongly affected by augmentation of the reflection coefficient, while the propagation coefficient computed by the RV-two point method is not affected by increasing reflection coefficient and still close to theoretical values. This finding confirms the interest of our method.

An other important result carried out from our work, shown in Figures 5a and 5b, is that the method is slightly sensitive to the effect of augmentation of wave reflection for the entire range of frequency investigated. The values of wave speed are close to theoretical values except for high reflection coefficient and frequency. Values of wave speed and attenuation vary between 0.999 and 1.001 times the theoretical values over the range of frequency corresponding to a variation of refection coefficient between $K = 0.06$ and $K = 0.96$, witch corresponds to physiological values of this coefficient in normal hemodynamic conditions. Several authors estimate that normal reflection coefficient is higher as 0.8 in femoral artery in normal condition, as reported by Westerhof et al. [38]. This value can be increased up to 0.95–1, in presence of vasoconstriction. Nichols and O'Rourke [39]. Li et al. [28] estimate that the reflection coefficient amplitude was in the range 0.3–0.7 at low frequencies, and in the range 0.1–0.3 at high frequencies.

The simulated results of phase velocity and attenuation plotted against the length of the tube — which corresponds to an equivalent position of the site of reflection — over the range of frequency is shown in Figure 6. They turn out the independence of computed propagation coefficient on the length of the tube. They clearly show that increasing tube length from $L = 0.16$ to $L = 0.96$ has no effect on the determination of both wave speed and attenuation. These values oscillated slightly about theoretical values only at high frequency. The magnitude of oscillations increase with increasing frequency but still lower than 1% for both phase velocity and attenuation. The length values used here correspond to many human arterial trees, as the femoral, tibial and subclavian artery as reported by Wang et al. [40], which make might our method to be useful for in vivo applications.

Figure 6a shows the normalised phase velocity over the range of frequency plotted as function of the tube length, the computed values are in good agreement with the theoretical one, and depend slightly on the frequency. The discrepancy is larger for attenuation as shown in Figure 6b and for all other data. This finding confirms the literature report [26,31]. The reason for this discrepancy can be explained by the method used to compute them: attenuation and wave speed are respectively the real and imaginary part of propagation coefficient, therefore; they will be calculated with the same absolute error, and the computed values of attenuation related to the imaginary part will be more affected than values of phase velocity.

In this work we had investigate by simulation, a new method for the determination of propagation coefficient, based on the non invasive measurement of velocity and radius at two sections of a tube or of an arterial tree. The effect of reflection and of the length of the tube have been studied at high and low wave speed. The results obtained using our RV-two point method for both phase velocity and attenuation are in a good agreement with the theoretical values over the range of frequency investigated. For noisy signals, attenuation shows a fluctuating behaviour only at high frequency. The amplitude of the fluctuations increases, with increasing the frequency values but still lower than 2% for phase velocity and 5% for attenuation for low frequencies ($F < 6$ Hz). The method is more precise for low frequencies corresponding to the first harmonics. The phase velocity and attenuation computed over the range of frequency, was in good agreement with theoretical values independently of reflection coefficient. The increase of reflection coefficient doesn't affect the accuracy of computed phase velocity. The source of the small discrepancy seems to be caused by numerical error due to use of Fourier analyse when the continuous spectrum is transformed into a discrete spectrum by the FFT algorithm, and the small amplitude of the higher components of the signal in the frequency range. We have also to recall that equation (15) is based on the hypothesis $\gamma_n d \ll 1$, with $\gamma = a + i\omega/C$ hypothesis which is not very realistic for high frequencies.

5 Conclusions

We presented in this paper a new method for estimating propagation coefficient from non invasive measurements of velocity and radius in two sites of an arterial tree.

In all cases studied, both attenuation and phase velocity computed by our method are in good agreement with theoretical values. Small deviations are shown at high frequencies when increasing reflection coefficient. The accuracy of the method decreases when increasing frequency. The discrepancies between theoretical and computed attenuation values are larger than those of phase velocity for the same physical and geometrical condition but are still very small. Nevertheless, the values of propagation coefficient carried from this method are slightly affected by noise. This method is quite simple which does not require heavy computational methods, and can be easily used in clinical environment. It is robust considering the noise of the signals. The results show also clearly that one takes advantage to use the low frequency range of the signal to improve the quality of the indirect measurement of the propagation coefficient.

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